

Global monopoles and massless dilatons in Einstein-Cartan gravity

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Abstract

A global monopole in dilatonic Einstein-Cartan gravity is presented. A linearized solution representing a global monopole interacting with a massless dilaton is found where Cartan torsion does not interact with the monopole Higgs field. Computation of the geodesic equation shows that the monopole-dilaton system generates a repulsive gravitational field. The solution is shown to break the linear approximation for certain values of torsion.

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Recently Garcia de Andrade and Barros [1] have found that the presence of global monopoles in linearized Einstein-Cartan gravity [2] lead to modified torsionic effects on physical quantities related to the global monopole such as the deficit angle and the density distribution which are very much in agreement with the predictions of COBE [3]. Earlier Dando and Gregory [4] have found the gravitational field of a global monopole within the context of low energy string gravity, where the global monopole couples with massive and massless dilatons. They found that for massless dilatons the spacetime is generically singular, whereas when the dilaton is massive the monopole induces a long range dilaton cloud. Since dilatons have been recently investigated in the context of Einstein-Cartan gravity [5] it seems interesting to extend their investigation to spacetimes with torsion. Let us consider the simplest model of Barriola-Vilenkin [6] to build global monopoles which is given by the Lagrangean

$$L(\psi^i) = \frac{1}{2} \nabla_a \psi^i \nabla^a \psi^i - \frac{\lambda}{4} (\psi^i \psi^i - \eta^2)^2 \quad (1)$$

where ψ^i is a triplet of real scalar fields, $i = 1, 2, 3$ and $a, b = 0, 1, 2, 3$ are spacetime indices. The model has a global $O(3)$ symmetry spontaneously broken to a global $U(1)$ symmetry by the vacuum choice $|\psi^i| = \eta$. Static spherically symmetric solutions are searched describing a global monopole at rest. The field configuration for this monopole is written according to Dando and Gregory as $\psi^i = \eta f(r) e^i$ where e^i is the unit radial vector in the internal space of the monopole. We consider the metric of the monopole as written by

$$ds^2 = B(r) dt^2 - A(r) dr^2 - r^2 d\Omega^2 \quad (2)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Since the linearized global monopole solution of Einstein-Cartan gravity in linear approximation has already been found we will not repeat it here, and go directly to the Global monopole in dilatonic Einstein-Cartan gravity. Its action is given by

$$S = \int d^4x (-g)^{\frac{1}{2}} [e^{-2\phi} (-R' - 4(\nabla\phi)^2 - V'(\phi)) + e^{2a\phi} L] \quad (3)$$

where L is the monopole Lagrangean for the monopole as given in (1), $V(\phi)$ is the potential for the dilaton. The action (3) is written in terms of the string metric of the string sigma model. We write the action in terms of the conformal metric

$$g_{ab} = e^{-2\phi} g'_{ab} \quad (4)$$

which turns the action into

$$S = \int d^4x (-g)^{\frac{1}{2}} [e^{-2\phi} (-R + 2(\nabla\phi)^2 - V(\phi)) + e^{2(a+2)\phi} L(\phi, e^{2\phi})] \quad (5)$$

The energy-momentum tensor is decomposed in three parts. The first is the monopole one

$$T_{ab}^m = e^{-2\phi} \nabla_a \phi^i \nabla_b \phi^i - g_{ab} L \quad (6)$$

the dilaton tensor

$$S_{ab} = 2\nabla_a \phi \nabla_b \phi + \frac{1}{2} V(\phi) - g_{ab} (\nabla\phi)^2 \quad (7)$$

and finally the torsion EMT

$$T_{ab}^{torsion} = 3S_{acd} S_b^{cd} - \frac{1}{2} g_{ab} S^2 \quad (8)$$

where $S^2 = S_{abc} S^{abc}$ is the square of the torsion tensor S_{abc} which S_{023} is the only non-vanishing torsion component chosen along the radial direction. The Einstein-Cartan equation can then be written in the quasi-Einsteinian form

$$G_{ab} = \frac{1}{2} e^{2(a+2)\phi} T_{ab}^m + S_{ab} + T_{ab}^{torsion} \quad (9)$$

Although the contribution of torsion in the problem considered here seems minor we shall prove that the perturbation method will produce a new torsion dependent global monopole in the Einstein-Cartan dilatonic gravity. Since there is no direct interaction between torsion and the monopole or the dilatonic field the dilaton equation will be the same as the one considered by Dando and Gregory. Once more we take $\eta\lambda^{\frac{1}{2}} = 1$ and the modified energy-momentum tensor is

$$T_t^t = \alpha(\gamma(\frac{f'^2}{2A} + \frac{f^2}{r^2}) + \frac{1}{4}\delta) \quad (10)$$

and

$$T_r^r = \alpha(\gamma(-\frac{f'^2}{2A} + \frac{f^2}{r^2}) + \frac{1}{4}\delta) \quad (11)$$

and

$$T_\theta^\theta = \alpha(\gamma\frac{f'^2}{2A} + \frac{1}{4}\delta) \quad (12)$$

where $\delta = (f^2 - 1)$, $\alpha = e^{2(a+2)\phi}$ and $\gamma = e^{-2\phi}$. The tt and rr components of Einstein-Cartan-dilaton-global monopole system now becomes

$$\frac{A'}{rA^2} + \frac{1}{r^2}\left(1 - \frac{1}{A}\right) = \epsilon T_t^t + \frac{1}{2}V(\phi) + \frac{\phi'^2}{A} + \frac{b}{r^2} \quad (13)$$

and

$$\frac{-B'}{rB^2A^2} + \frac{1}{r^2}\left(1 - \frac{1}{A}\right) = \epsilon T_r^r + \frac{1}{2}V(\phi) - \frac{\phi'^2}{A} + \frac{b}{r^2} \quad (14)$$

where we have chosen the Barros ansatz for torsion of the global monopole as $S^2 = \frac{(1-b)\eta^2}{r^2}$ where b is a constant which when zero reduces the solution to the Barriola- Vilenkin solution in the absence of dilatons. To simplify matters we consider in this Letter just massless dilatons leaving the massive dilatons case for a future paper. For massless dilaton Einstein- Cartan gravity $V(\phi) = 0$, where $\epsilon = \frac{\eta^2}{2}$ and making the expansion

$$A = 1 + \epsilon A_1 + \dots \quad (15)$$

and

$$B = 1 + \epsilon B_1 + \dots \quad (16)$$

and

$$\phi = \phi_0 + \epsilon \phi_1 + \dots \quad (17)$$

To the first order we have, $A' = B' = 0$ and the equations reduce to

$$\phi_0'^2 - \frac{b}{r^2} = 0 \quad (18)$$

which yields the simple solution $\phi_0'^2 = b^{\frac{1}{2}} \ln r$. Substitution of these values into fields equations we obtain

$$\frac{A'_1}{r} + \frac{A_1}{r^2} = \beta r \left(\frac{f'^2}{2} + \frac{f^2}{r^2} \right) + \frac{1}{4} \beta r (f^2 - 1)^2 + \frac{b}{r^2} (A_1 - 1) \quad (19)$$

and

$$\frac{-B'_1}{r} + \frac{A_1}{r^2} = \beta r \left(-\frac{f'^2}{2} + \frac{f^2}{r^2} \right) + \frac{1}{4} \beta r (f^2 - 1)^2 + \frac{b}{r^2} (A_1 + 1) \quad (20)$$

$$\phi_1'' + \frac{2\phi_1'}{r} = (a+1) \frac{\beta}{r} \left(-\frac{f'^2}{2} \right) + \frac{a+2}{4} (f^2 - 1)^2 \quad (21)$$

For large r , f is approximately unity which greatly simplifies the system to

$$\phi_1'' + \frac{2\phi_1'}{r} = 0 \quad (22)$$

which can be solved to yield

$$\phi_1' = er - \frac{2c}{r^3} + d \quad (23)$$

where here we are using $\beta = e^{2(a+1)\phi_0}$. The field equations for the metric are

$$\frac{A_1'}{r} + \frac{A_1}{r^2} = \frac{\beta}{r} + \frac{b}{r^2}A_1 \quad (24)$$

and

$$\frac{-B_1'}{r} + \frac{A_1}{r^2} = \frac{\beta}{r} + \frac{b}{r^2}A_1 \quad (25)$$

Their solution is

$$A_1 + B_1 = 0 \quad (26)$$

Substitution of the solutions of these equations on the expansion we obtain the following final metric

$$ds^2 = \left(1 - \frac{\epsilon\beta r}{2(b-1)} - \epsilon\frac{m}{r}\right)dt^2 - \left(1 + \frac{\epsilon\beta r}{2(b-1)} + \epsilon\frac{m}{r}\right)dr^2 - r^2d\Omega^2 \quad (27)$$

where m is an integration constant. This implies the following geodesics

$$\dot{v}_r = \frac{\beta}{2(b-1)} + \frac{m}{r^2} \quad (28)$$

which very far from the sources shows that the dilaton-global monopole-torsion system is gravitationally repulsive. Note that the torsion factor b is determinant for the repulsive character of the global monopole-dilaton system. Notice that due to the presence of βr . Therefore the spacetime should be bound to values of $r < (b-1)$ and the need for an exact solution is of prime importance for the better understanding of the problem. A more detailed account of the ideas discussed here may appear elsewhere.

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